1 Number System

Learning Outcomes

- Decimal Number System
- Binary Number System
- Octal Number System
- Hexa Number Decimal System
- Computer Arithmatic

In the earlier time, when there were no means of counting, people used to count with the help of fingers, stones, pebbles, sticks, etc. These methods were not adequate and had many limitations. To overcome these limitations, many number systems were introduced with the passage of time, like:

- Decimal Number system
- Binary number system
- Octal number system
- Hexadecimal number system

A number system is a set of values used to represent different quantities, such as number of students in a class, number of viewers watching a particular show, etc. We use the decimal number system in our day life, daily lives whereas a computer represents all kinds of data and information (text, numbers, graphics, etc.) in the binary number system.

The total number of digits used in a number system is called its **Base** or **Radix**. The base is written after the number as a subscript, for example, $(15)_{10}$ which mean a decimal number.

Decimal Number System

The need for counting paved the way to introduce the Decimal number system, in which 0,1,2,3 .... 9 are used to form any number. Most of our arithmetic operations are performed with decimal numbers.

Decimal number system consists of ten digits, i.e ., 0 to 9 with the base 10. Each number can be used individually or they can be grouped to form a numeric value, e.g ., 79, - 89, 53.8, - 88.53, etc. The value of each digit in a number depends upon the following :

- The face value of the digit.
- The position of the digit in the number.
- The base of the number system.

Each position represents a specific power of the base(10). The right most digit of a number is called **Least Significant Digit**, whereas the left most digit is called **Most Significant Digit**. For example, the number 689 can be represented in the following way:

$9*10^0$ units = 9
$8*10^1$ tens = 80
$6*10^2$ hundreds = 600
689

Observation

The positional value of each digit increases ten folds as we move from right to left. In the above mentioned example, 6, 8, and 9 are the face values and their place values are hundreds, ten, and units, respectively. The place value of a digit depends on its position in the number. Now, we will discuss the various types of numbers that are used in a computer.

Binary Number System

The Binary number system consists of only two digits, i.e ., zero and one (0 and1). Since this system uses two digits, it has the base 2. All digital computers use this number system and convert the input data from the decimal format into its binary equivalent.

Why Binary?

A computer cannot understand human language, rather it understands only the binary code. Therefore, the data that is entered into a computer is converted into its binary equivalent. It further converts the binary result into its decimal equivalent to generate an output.

Conversion of Decimal into Binary Number System

The equivalence between binary and decimal numbers can be understood with the given examples.

To convert a decimal number into a binary number, follow the given steps:

Step 1: Divide the given decimal number with the base 2.

Step 2: Write down the remainder and divide the quotient again by 2.

Step 3: Repeat the step 2 till the quotient is zero.

Step 4:Write the remainders obtained in each step in the reverse order to form the binary equivalent of the given decimal number, i.e., placing the Least Significant Digit at the top and the Most Significant Digit at the bottom.

Consider the following example:

Example 1: $36 = \_\_\_\_\_{}_2$

2	36	
2	18	– 0
2	9	– 0
2	4	– 1
2	2	– 0
2	1	– 0
2	0	– 1

$36 = (100100)_2$

Example 2: $45 = \_\_\_\_\_{}_2$

2	45	
2	22	– 1
2	11	– 0
2	5	– 0
2	2	– 1
2	1	– 0
2	0	– 1

$45 = (101001)_2$

Observe, that the remainders obtained each step are written in the reverse order.

Conversion of Binary to Decimal Number System

To convert a binary number into its equivalent decimal number, follow these steps:

Step1: Multiply each binary number with its positional value, which is in terms of power of 2, starting from the extreme right digit.

Step 2: Increase the power one by one, keeping the base fixed as 2.

Step 3: Calculate the sum of all the products to get the decimal number.

Example 1: $1101110_2 = \_\_\_\_\_$

1101110

$0 \times 2^0 = 0$
$1 \times 2^1 = 2$
$1 \times 2^2 = 4$
$1 \times 2^3 = 8$
$0 \times 2^4 = 0$
$1 \times 2^5 = 32$
$1 \times 2^6 = 64$
110

$(1101110)_2 = 110$

Example 2: $11001_2 = \_\_\_\_\_$

11001

$1 \times 2^0 = 1$
$0 \times 2^1 = 0$
$0 \times 2^2 = 0$
$1 \times 2^3 = 8$
$1 \times 2^4 = 16$
25

$(11001)_2 = 25$

Example 3: $1000110_2 = \_\_\_\_\_$

$= 1000110$ (6 5 4 3 2 1 0)

$= 1X2^6 + 0X2^5 + 0X2^4 0X2^3 + 1X2^2 + 1X2^1 + 0X2^0$

$= 64^0 + 0 + 0 + 4 + 2 + 0$

$= 70$

$(1000110)_2 = 70$

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Octal Number System is commonly used as a shorter representation of binary numbers by grouping binary digits into threes.

Octal Number System

The Octal number system (oct) consists of 8 digits, from 0 to 7, with the base 8. The concept of Octal number system came from the Native Americans as they used to count numbers by using the space between their fingers rather than using their fingers. The procedure of 'Octal to Decimal' conversion is similar to 'Binary to Decimal' conversion. The only difference is the change of base.

To convert an octal number to decimal number, start multiplying the digits of the number from the right-hand side with increasing power of 8 starting from 0 and finally calculating all the products.

Example 2: $(672)_8$

6 7 2

$2 \times 8^0 = 2$
$7 \times 8^1 = 56$
$6 \times 8^2 = 384$
442

$(672)_8 = 442$

Example 3: $(246)_8 = \_\_\_\_\_$

$= 2x8^2 + 4x8^1 + 6x8^0$

$= 128 + 32 + 6$

$= 166$

$(246)_8 = 166$

Hexa Decimal Number System

This number system consists of 16 digits, numbers 0-9 and the letters A-F represent decimal numbers from 10 to 15. That means, A is equivalent to 10, B is equivalent to 11, C refers to 12, and so on. The base of this number system is 16. This number system is also known as Hex. The procedure of converting hexadecimal to decimal is similar to the methods shown in the previous pages, with the only difference being the change of base.

To convert a hexadecimal number to decimal, multiply the numbers starting from the right-hand side with increasing power of 16, starting from 0 and then calculating the sum of all the products.

Example 4: $A41_{16}$ = _____

$A41_{16}$

10

$= 10 \times 16^2 + 4 \times 16^1 + 1 \times 16^0$

$= 2560 + 64 + 1$

$= 2625$

$(A41)_{16} = 2625$

Example 5: $C2D_{16}$ = _____

$C\ 2\ D_{16}$

$12\ \ 13$

$= 12 \times 16^2 + 2 \times 16^1 + 13 \times 16^0$

$= 3072 + 32 + 13$

$= 3117$

$(C2D)_{16} = 3117$

Computer Arithmetic

As a computer understands only the binary code, so the data entered into the computer by the user is converted into binary code for processing. This processing may involve various kinds of arithmetic operations, such as addition, subtraction, multiplication, division, etc ., on the binary numbers.

Binary Addition

The technique used to add binary numbers inside the computer is very easy and simple. This is performed in the same way as you perform addition with decimal numbers.

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While adding 1 + 1, the output will be 10, where 0 is written under the same column and carry over 1 is shifted to the next place, as it happens in the decimal number addition.

a	b	a + b = c
0	0	0 + 0 = 0
0	1	0 + 1 = 1
1	0	1 + 0 = 1
1	1	1 + 1 = 10

Example 6:

$11001_2 + 1101_2$

1 ← carry over

1 1 0 0 1

+ 1 1 0 1 CARRY OVER

100110_2

Example 7:

$111011_2 + 110011_2$

→ 1 1 1 1

1 1 1 0 1 1

+ 1 1 0 0 1 1

1101110_2

Binary Subtraction

The rules given in the table must be followed to perform binary subtraction:

a	b	a – b = c
0	0	0 – 0 = 0
1	0	1 – 0 = 1
1	1	1 – 1 = 0
0	1	0 – 1 = 1

Example 8:

$110011_2 - 10010_2$

1 1 0 0 1 1

– 1 0 0 1 0

100001_2

Example 9:

$10100_2 - 101_2$

1 10 1 10

10 1

1 0 1 0 0

– 1 0 1

1111_2

Binary Multiplication

The rules for performing multiplication using binary numbers is same as that of the decimal numbers.

The given table illustrates the multiplication of two binary digits:

a	b	a * b = c
0	0	0 * 0 = 0
0	1	0 * 1 = 0
1	0	1 * 0 = 0
1	1	1 * 1 = 1

Example 10:

$11_2 \times 10_2$

$$\begin{array}{r} 1\ 1 \\ \times\ 1\ 0 \\ \hline 0\ 0 \\ +\ 1\ 1\ \times \\ \hline 1\ 1\ 0_2 \end{array}$$

Example 11:

$1011_2 \times 101_2$

$$\begin{array}{r} 1\ 0\ 1\ 1 \\ \times\ 1\ 0\ 1 \\ \hline 1\ 0\ 1\ 1 \\ 0\ 0\ 0\ 0\ \times \\ 1\ 0\ 1\ 1\ \times \\ \hline 1\ 1\ 0\ 1\ 1\ 1_2 \end{array}$$

Binary Division

The method to perform division of two binary numbers is same as that of decimal numbers. See the example given below :

a	b	a ÷ b = c
0	0	0 ÷ 0 = 0
0	1	0 ÷ 1 = 0
1	0	1 ÷ 0 = 0
1	1	1 ÷ 1 = 1

Example 12:

$111_2 \div 10_2$

Divisor → 10 | 111 ← | 11 ← Quotient

− 10 | → Divident

11

10

1 → Reminder

Example 13:

$101110011_2 \div 1101_2$

Divisor → 1101 | 101110011 ← | 11100 ← Quotient

− 11011 | → Divident

10100

1101

1110

1101

0111

0000

111 → Reminder

Quick Look

- A number system is a set of values used to represent different quantities, such as number of students in a class, number of viewers watching a particular show, etc.
- Decimal number system consists of ten digits, i.e ., 0 to 9 with the base 10, e.g. 79, - 89, 53.8, - 88.53.
- The right most digit of a number is called Least Significant Digit, whereas the left most digit is called Most Significant Digit.
- The Binary number system consists of only two digits, i.e ., zero and one (0 and1) and it has the base 2.
- The data that is entered into a computer is converted into its binary equivalent. It further converts the binary result into its decimal equivalent to generate an output.
- The Octal number system consists of 8 digits, i.e ., 0 to 7, with the base 8.
- Octal number system came from the Native Americans as they used to count numbers by using the space between their fingers rather than using their fingers.
- Hexadecimal number system consists of 16 digits, numbers 0-9 and the letters A-F represent decimal numbers from 10 to 15.
- Computer processing may involve various kinds of arithmetic operations, such as addition, subtraction, multiplication, division, etc ., on the binary numbers.
- Binary Addition is used to add binary numbers inside the computer which is very easy and simple.
- The rules for performing multiplication using binary numbers is same as that of the decimal numbers.

Brain Booster

Section - I

Objective Type Questions

A. Fill in the blanks with the correct words.

1	Computer System	16	10	Octal

1. _____ is the base of Decimal number system.
2. The base of _____ number system is 8.
3. Hexadecimal number system consists of _____ digits.
4. In binary multiplication, 1*1 equals to _____.
5. Binary number system is understood by the _____.

B. Write T for the true statement and F for the false one:

1. You can perform arithmetic operations on binary numbers.
2. The octal number system consists of 8 digits, i.e ., 1 to 8.
3. The base of Hexadecimal Number system is 8.
4. 1 subtracted from 0 equal to 1.
5. Aryabhata introduced the concept of 0 (zero).

C. Choose the right option:

1. _____ introduced the concept of 0 (zero).

   a. Ada lovelace
   b. Aryabhata
   c. Charles Babbage
   d. None of these

2. The decimal format is converted to its binary equivalent by _____.

   a. Digital Computer
   b. Cell phone
   c. Abacus
   d. All of these

3. Only the _____ code is understood by a computer.

   a. English
   b. French
   c. Binary
   d. None of these

4. In binary addition, 1+1 equals to _____.

a. 0 ◯ b. 10 ◯

c. 2 ◯ d. 1 ◯

5. To convert a decimal number into a Hexadecimal, divide the number by _____.

a. 2 ◯ b. 8 ◯

c. 16 ◯ d. Both (a) and (b) ◯

D. Application Based question

- Savya's computer teacher has asked her to convert a Binary to Decimal number. Suggest the method which she should apply in converting the Binary number.
- The teacher has given an assignment to Brishkettu on binary multiplication. Brishkettu is confused about how to multiply 1 with 0. Help him in solving the problem.

Section - II

▶ Descripctive Type Questions

E. Answer the following question.

1. Explain what is a Number system. Also mention its types.

_____

_____

2. What are the rules to convert a decimal number into an octal number?

_____

_____

3. Write the rules to add two binary numbers.

_____

_____

4. Briefly explain the Binary number system.

_____

_____

5. What is a hexadecimal number system? Explain.

_____

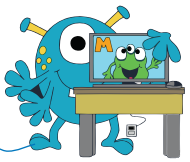
_____

Activity TIME

Activity Time

LAB ACTIVITY

Practice Makes Perfect



- Convert the following Decimal numbers

  a. 174 b. 13 c. 415

- Convert the following Binary numbers

  a. 1100 b. 1001111 c. 1010001

- Perform Binary addition on the following:

  a. 11101+11011 b. 10111+110001 c. 10001+ 11101

- Find the difference between the following

  a. 1011101 - 111100 b. 1101010 – 101101 c. 101-001

- Multiply the following Binary numbers:

  a. 100 x 011 b. 1010 x 1011 c. 10101x 101

PROJECT WORK

For Practicing Ideas

- Make a presentation on Binary Number System. Set a beautiful background, apply nice formatting and add animation effects to it.

TEACHERS NOTE

smart suggestions

- Please explain number system with the help of simple day to day examples.
- Explain the various types of number systems with their uses.
- Explain what is Computer Arithmetic.

ONLINE LINKS

For Searching More

- http://byjus.com/maths/number-system/
- http://www.vedantu.com/maths/number-system/
- http://www.toppr.com/guides/computer-aptitude-andknowledge/basics-of-computers/number-system/